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Proof of hypothesis of twin primes

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ABSTRACT

1. Find the distributive law of primes,

2. By the distributive law of primes, we may prove Hypothesis of twin primes,

$$T_{2m+1\sim 4m+1} \geqslant T_{inf} = \left[\frac{\left(\frac{4m+1}{\log(4m+1)}\alpha - \frac{2m}{\log 2m}\beta\right)^2}{2m+2}\right] + 1 \geqslant 1, \quad (5 \leqslant m < \infty).$$

3. By the integration we may obtain the most simple expression of T_{inf},

$$T_{0\sim 2m} \geqslant T_{inf} = \left[\frac{2m}{(\log 2m)^2}\right] + 1 > 1, \quad (6 \leqslant m < \infty).$$

Keywords: Hypothesis of twin primes; prime; infimum; distributive law.

SUBJECT CLASSIFICATION MSC (2010) 11A41. MSC (2010) 11N 05.

1.Theorem 1. The distributive law of primes in natural numbers,

$$\left(\frac{x}{\log x}\right)\alpha < \pi(x) \leqslant \left(\frac{x}{\log x}\right)\beta, \quad (11 \leqslant x < \infty). \tag{1}$$

Proof. To found an exponential function of density of odd primes,

$$y = x^{\frac{\pi(x)}{x}}, \quad (3 \leqslant x < \infty).$$

$$\text{then} \quad \pi(x) = \frac{x}{\log x}\log y,$$

$$\because \quad \lim_{x\to\infty}\frac{\pi(x)}{x} = \lim_{x\to\infty}\frac{1}{\log x},$$

$$\therefore \quad \lim_{x\to\infty} x^{\frac{\pi(x)}{x}} = \lim_{x\to\infty} x^{\frac{1}{\log x}},$$

$$\because \quad x^{\frac{1}{\log x}} = e, \quad (3 \leqslant x < \infty).$$

$$\therefore \quad \lim_{x\to\infty} x^{\frac{\pi(x)}{x}} = e = y_{min},$$

$$\log y_{min} = 1 = \alpha,$$

when $x \geqslant 11$, $y_{min} < y \leqslant y_{max}$,

$$\log y_{max} = \log 113^{\frac{30}{113}} = \beta.$$

$$x \log y_{min} < x \log y \leqslant x \log y_{max}, \quad (11 \leqslant x < \infty).$$

$$\left(\frac{x}{\log x}\right)\alpha < \pi(x) \leqslant \left(\frac{x}{\log x}\right)\beta, \quad (11 \leqslant x < \infty).$$

(1) is obtained.

Theorem 1 is proved.

2.Theorem 2. The number of twin primes is infinite.

$$T_{2m+1\sim 4m+1} \geqslant T_{inf} = \left[\frac{\left(\frac{4m+1}{\log(4m+1)}\alpha - \frac{2m}{\log 2m}\beta\right)^2}{2m+2}\right] + 1 \geqslant 1, \ (5 \leqslant m < \infty). \quad (2)$$

Proof. Let $T_{2m+1\sim 4m+1}$ be the number of twin primes in the interval

$[x = 2m+1, 4m+1]$,

When $m \geqslant 5, (x \geqslant 11)$ is given.

1. To found an odd mathematical model:

$\{ 2m+1, 2m+3, \cdots, \cdots, 4m-1, 4m+1 \}$

$\{ 2m+1, 2m+3, \cdots, \cdots, 4m-1, 4m+1 \}$

The difference of two meeting numbers $= 2$,

The number of odd numbers $= 2m+2$.

The average value of $T_{2m+1\sim 4m+1}$, i.e. T_{av},

$$T_{av} = \frac{(\pi(4m+1) - \pi(2m))^2}{2m+2}, \quad (5 \leqslant m < \infty). \quad (A)$$

2. By (1), transforming (A) into the infimum of $T_{2m+1\sim 4m+1}$, i.e. T_{inf},

(decreasing minuend, increasing subtrahend)

$$T_{2m+1\sim 4m+1} \geqslant T_{inf} = \left[\frac{\left(\frac{4m+1}{\log(4m+1)}\alpha - \frac{2m}{\log 2m}\beta\right)^2}{2m+2}\right] + 1, \ (5 \leqslant m < \infty).$$

$m = 5$, $[11, 21]$, $T_{2m+1\sim 4m+1} > 1$, $T_{inf} = 1$,

$m = 6, 7, 9, 10$, $T_{2m+1\sim 4m+1} = T_{inf} = 1$,

four critical points of T_{inf}.

The ladder-like line of T_{inf}:

$$m=5\sim 43, \quad T_{inf}=1,$$

$$m=44\sim 107, \quad T_{inf}=2,$$

$$\because \quad T_{2m+1\sim 4m+1}\geqslant 1, \quad (5\leqslant m<\infty).$$

$$T_{2m+1\sim 4m+1}\geqslant 1, \quad (1\leqslant m\leqslant 5).$$

$$\therefore \quad T_{2m+1\sim 4m+1}\geqslant 1, \quad (1\leqslant m<\infty).$$

(2) is obtained.

Theorem 2 is proved.

3.Theorem 3. By the integration we may obtain the most simple expression of T_{inf},

$$T_{0\sim 2m}\geqslant T_{inf}=[\frac{2m}{(\log 2m)^2}]+1>1, \quad (6\leqslant m<\infty). \qquad (3)$$

Proof. Let $T_{0\sim 2m}$ be the number of twin primes in the interval [0,x=2m],

when m⩾6,(x⩾12) is given.

1. To found an odd mathematical model, it contains all odd numbers of [0,2m]:

| 1 , 3 , ⋯ , ⋯ , 2m−3 , 2m−1 |

| 1 , 3 ,⋯ , ⋯ , 2m−3, 2m−1 |

The difference 0f two meeting numbers=2,

The number of odd numbers=2m.

Referring to (A), due to this quadratic form of the numerator,

$$(\pi(4m+1)-\pi(2m))^2,$$

we assume $d(T_{0\sim 2m})=2\lambda\, d\lambda$,

The $d\lambda$ indicates that λ is the variable of integration.

$$\text{then} \quad T_{0\sim 2m}==\int_{\pi(0)}^{\pi(2m)} 2\lambda\, d\lambda,$$

Find the average number of $T_{0\sim 2m}$, i.e. T_{av},

$$T_{av}=\frac{1}{2m}\int_{\pi(0)}^{\pi(2m)} 2\lambda\, d\lambda,$$

$$=\frac{1}{2m}[\lambda^2]_{\pi(0)}^{\pi(2m)}$$

$$=\frac{1}{2m}((\pi(2m))^2-(\pi(0))^2), \qquad (B)$$

2. By (1), transforming (B) into the infimum of $T_{0\sim 2m}$, i.e. T_{inf},

(decreasing minuend, increasing subtrahend)

$$T_{inf}=\left[\frac{\left(\frac{2m}{\log 2m}\alpha\right)^2-((0)\beta)^2}{2m}\right]+1=\left[\frac{2m}{(\log 2m)^2}\right]+1$$

$=[h(m)]+1$, $(6\leqslant m<\infty)$.

$m=6$, [0,12], $T_{0\sim 2m}>1$, $T_{inf}>1$,

$m=6$, $T_{0\sim 2m}=T_{inf}=2$,

$m=7,8,9$, $T_{0\sim 2m}=T_{inf}=3$,

four critical points of T_{inf},

The ladder-like line of T_{inf}:

$m=6$, $T_{inf}=2$,

$m=7\sim 20$, $T_{inf}=3$,

...

The characteristics of T_{inf}:

①. Uniformly continuous. h(m) is an elementary function, its interval of definition [6, m] is closed, thus, $[h(m)]+1$ is uniformly continuous in the interval [6, m].

②. Monotone increasing. Differentiating the function h(m),

$$h'(m)=\frac{2(\log 2m-2)}{(\log 2m)^3}>0,\quad (6\leqslant m<\infty).$$

$[h(m)]+1$ is monotone increasing in the interval [6, m].

$$T_{0\sim 2m}\geqslant T_{inf}=\left[\frac{2m}{(\log 2m)^2}\right]+1>1,\quad (6\leqslant m<\infty).$$

3. ∵ $T_{0\sim 2m}>1$, $(6\leqslant m<\infty)$.

$T_{0\sim 2m}\geqslant 1$, $(3\leqslant m\leqslant 6)$.

∴ $T_{0\sim 2m}\geqslant 1$, $(3\leqslant m<\infty)$.

(3) is obtained.

Theorem 3 is proved.

The number of twin primes is infinite.

REFERENCES

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